Universitat Autònoma de Barcelona

European Music Portfolio – Maths:
Sounding Ways into Mathematics (EMP-M)

Booklet of back-up material for teachers

Coordinated by:
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On behalf of:
MUSICOMÀTICS

(EMP-M working group at Catalonia)

Lifelong Learning Programme

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1. Introduction

The material presented in this booklet is a tool intended to support teacher training carried out within the framework of the *European Music Portfolio: Sounding Ways into Mathematics* (EMP-M), a project which, as explained in greater detail below, seeks to explore the relationship between **music and mathematics** in order to contribute to a **more holistic education** (in the sense of Viladot & Cslovjecsek, 2015). What you will find here is fruit of the contrast of **theory**, gathered together and discussed by experts in music and maths teaching, and **practice**, applied and discussed by active teachers and teacher trainers. Therefore, the proposed innovations can lay claim to a solid theoretical foundation (see Casals, Carrillo & González-Martín, 2014; Viladot et al., in press) that has been complemented by experimental trials in the world of education through its application in schools in the various countries participating in the project, which acted as pilot centres[1].

The EMP-M is a three-year Comenius project (2013-2016), part of the *Lifelong Learning Programme*. Led by the *University of Applied Sciences Northwestern Switzerland* (FHNW), nine institutions in seven different countries (Switzerland, United Kingdom, Spain, Czech Republic, Romania, Greece and Germany) took part. The antecedents to the project lie in the international PRIME network[2]– which is now one of the ISME *Special Interest Groups* – and a previous Comenius project, the *European Music Portfolio: A Creative Way into Languages* (2009-2012)[3]. In these transnational forums work was and is based on the following principles: a) music is essential for personal development; b) as a form of artistic expression, music enhances individual creativity; c) music is a means of expression and communication that favours the learning of any subject; and d) education must go beyond the disciplinary compartmentalization of the current system (Ludke & Heinmann, 2012).

During its implementation, the EMP-M has focused its attention on the didactic relations linking music and mathematics and has explored new possibilities for the integration of the two subjects. This has led to the development of activities, materials and strategies addressed to primary school teachers. These resources have been created with the idea of establishing an interdependent relationship between music and mathematics, creating synergies that have a positive impact on both subjects rather than generating situations of dependency. Consequently, this material is useful for both music and mathematics teachers. Furthermore, the benefits of this material should be reflected by the results obtained in both subjects and, above all, by its impact on pupils' global education.

[1] For further information about the theoretical foundations of the projects and the training and practical activities designed for application in schools, see also Carrillo, Viladot, González-Martín and Casals (*in press*).

[2] *Practice and Research in Integrated Music Education*: http://www.sigprime.net/

[3] See Viladot and Casals (2012) or the website https://www.emportfolio.eu

Indeed, with regard to the pupils the project also aims to be helpful in two problematic areas shared at a European level: on the one hand, the apathy and poor maths performance in evidence among part of the school population, and on the other hand, the low esteem which music education has historically been held in both socially and, more specifically, on the school curriculum.

First of all, this booklet provides a brief introduction to the project and especially to how we understand the integration of music and mathematics in schools. Below, five activities illustrating the project are presented as inspirational material for those teachers who believe, like us, that it is important to work on music and mathematics together. These activities are intended to illustrate how an integrated approach can be used to study the basics of mathematics and music, always working from the standpoint of competence and practical experience. At the end, some links and references have been included for those looking for more resources or wishing to go deeper into the subject.

Ultimately, this booklet is intended to encourage and support creativity among teachers and provide a basis for a global, innovative approach to teaching and learning.

2. Applying the philosophy of the EMP-M project in the classroom

It is well known that there are many connections between the two disciplines. In fact, since the first contributions made by Pythagoras no one has questioned the close relationship between music and mathematics. Down through history, musicians, scientists and scholars have studied and drawn attention to the broad base that unites the two fields, with some aspects providing an opportunity to work more effectively within the framework of compulsory education, and not so much on the level of inter-related content (such as the relationship between fractions and the note values, or between dance and geometry or position in space), but especially in the area of the acquisition of competencies. To take an example, can we enhance our understanding of music as part of an improvement in reasoning and problem solving? In the EMP-M project we have worked in this direction.

The EMP-M project is designed from the standpoint of a holistic, creative and experiential approach to education, and it assumes the idea that schooling must be based on reality – in all its complexity – in order to finally achieve a global understanding of the environment. In other words, we advocate avoiding rigid education systems based on marked disciplinary compartmentalization in order to develop educational contexts in which learning is focussed on practical experience, activities and materials that promote creativity and reflection, as well as favouring the integrated understanding of the various disciplines (in the sense propounded by Viladot and Cslovjecsek, 2014).

Many authors have discussed the diversity existing in relation to interdisciplinarity and the levels of integration of subjects in the curriculum. The classification made by Russell Bowie (2009) is particularly useful. It distinguishes three models: a) when a subject is used to support another one (*service connections*); b) when common material is used to achieve the goals of each subject (*symmetric correlations*); and c) when the two subjects work together, seeking synergies to understand something different while achieving the goals of each subject and even more general ones (*syntegration*). In our opinion, true integrated learning occurs when working with this latter model.

It seems obvious that new ways of looking at things often emerge when the teaching and learning process combines different subjects. We use various visions and strategies for solving problems and obtaining a better understanding of what is being studied. According to Barrett (2001: 27), "deep understanding often depends on the interactions and intersections between the disciplines." On the other hand, we are aware that this often means abandoning methodologies where we feel confident as teachers – but which we think could be improved – in order to explore these new possibilities that we suspect are more effective.

In this project, through our training courses and through activities such as those presented in this booklet, we invite you to take the step of discovering how to work on music and mathematics simultaneously and creatively by drawing on simple but didactically effective and inspiring ideas and activities, adaptable to different educational levels and possessing great learning potential. Additionally, we believe that these proposals provide positive experiences for students who may feel sceptical about mathematics or musical activities. In our experience, this approach helps to overcome existing difficulties by giving the participants the space they need to feel more confident about the use of musical and mathematics competencies.

All in all, in this project we consider that the relationship established between music and mathematics must be focused on interaction, cooperation and, in particular, reciprocity between the content under study. We do not conceive the relationship between the two areas as juxtaposition, nor as a sum, but as a necessary interrelationship.

Accordingly, we decided to design and develop activities using an integrated teaching approach where music and mathematics are treated on an equal footing, together in the same classroom:

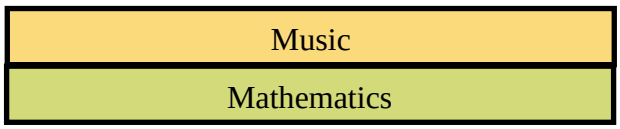


Fig 1: Integrated didactic model

The trials carried out in Catalonia in pilot schools show that, although this model is possible, the reality of schools, at least locally, often makes it difficult to put into practice owing to incompatibilities in the teachers' timetables and a lack of space in schools. This results in a split programme of activities, in which joint work ends up serving as an excuse to work on music and mathematics separately.

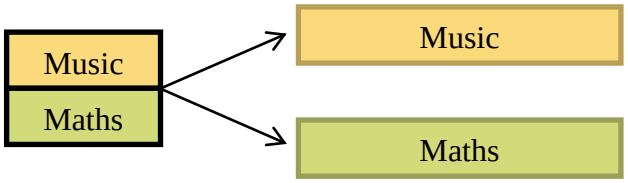


Fig 2: Split didactic model

It is necessary to prevent this from happening and, if it is difficult to put into practice an integrated model, a semi-integrated version should be the objective, which ensures points of connection between the two subjects throughout the implementation of the activity, or makes the contents evolve to meet again at some stage and generate a model presenting

maximum integration and as many connection points as possible. On the other hand, this should not be confused with situations where one of the subjects may act as a means to an end, i.e. in favour of the other (this would be a service relationship – *a service connection* – as described above). Both subjects should be treated equally, on the same level, with the pupils learning content from the two fields.

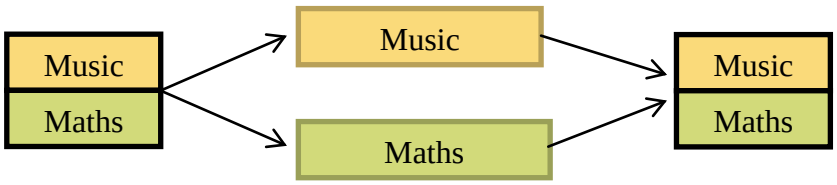


Fig 3: Semi-integrated didactic model

With the onus on coordination, dedication and planning, we are aware that devising and implementing this type of activity is not an easy task. To achieve this Castañer and Trigo (1995) agree that the teachers' attitude is fundamental and affirm that such activities require determination and willingness on the part of the teaching staff. However, as we said initially, the results compensate the effort because pupils are offered a type of teaching and learning that breaks down the boundaries between the disciplines, providing a more holistic vision of reality and motivating them to learn (González-Martín, 2013).

3. Examples of activities

Activities	MUSIC	MATHEMATICS
	COMPETENCIES **learning contexts** and key contents	COMPETENCIES **block of contents** and key contents
Let's dance to geometry!	LISTENING AND PERFORMING **Dance**: positions and dance steps **Listening**: structure	REPRESENTATION AND CONNECTIONS **Space and form**: square (sides, parallel sides, corners, diagonals, central point)
What's the LCM of the numbers 2, 3 and 5? Let's beat it out!	LISTENING AND PERFORMING **Body percussion**: rhythmic reading, beat, rhythmic regularity and precision	REASONING, TESTING, REPRESENTATION AND CONNECTIONS **Numbering and calculation**: multiplication, multiples, least common multiple
Obwisana	LISTENING AND PERFORMING **Singing and voice**: performing a song with gestures	SOLVING PROBLEMS, REASONING AND TESTING **Numbering and calculation**: multiplication, least common multiple
Making angles while moving to the music	LISTENING (AND PERFORMING) **Listening**: structure and phrasing **Dance and moviment:** expression and synchronisation	REPRESENTATION **Space and form:** angles and simmetry
What's the value of half a mínim? Fractions and rhythmic notation	LISTENING, PERFORMING AND COMPOSING **Singing:** performance, equivalence rhythmic figures **Listening**: listening, rhythmic notation	PROBLEM-SOLVING, REPRESENTATION AND CONNECTIONS **Numbering and calculation:** fractions (as part of a whole)

Let's dance to geometry!

Original idea: Albert Casals and Montserrat Prat

This activity, which brings together music and mathematics, consists of working on geometry while doing dance.

Competencies

- Aspects of mathematical competency: representation and connections.
- Aspects of musical competency: listening and performance.

Key points

- Mathematics: square, sides of a square, parallel sides, corners, diagonals.
- Music: dance steps, dance figures (arrangements), phrasing, structure, beat.

Description of the activity

1. This is a dance from the Patum festival, which is held in the town of Berga[4]. It is called Nans Vells and it is usually danced in squares formed by four people. In this case, the pupils try to make a square with 16 people. We're going to look at how we can do it.
2. What is the result? What problems will you encounter? How can you find solutions? Is it possible to create a square with 16 people?
3. With the "empty" square with 16 people (one at each corner and three per side), the pupils will learn the proposed dance (see Fig. 1).
4. We listen to the music. What do you notice about the music? What parts can they pick out? Pupils should notice the change of metre and tempo in the two sections.
5. Explanation of the dance:
    a. You must explain that each square has two pairs of parallel sides of equal length and all corners right angles. The first pair of parallels will be called A, and the second, B (see Fig. 1).
    b. During the first phrase (eight bars) the A parallels advance into the middle (four bars) and then go back to their places without turning round (four more bars). The corners should keep still or can turn round slowly on the spot (see Fig. 2).
    c. During the second phrase, the B parallels repeat the same movement.
    d. In the following two phrases, movements b and c are repeated.

[4] It can be listened in: https://www.youtube.com/watch?v=kT3LemEylFw

e. With the change of metre (second part), the corners join in. After discussing exactly what diagonals are, we imagine the two diagonals crossing the square. One will be *Diagonal 1*, and the other, *Diagonal 2*.
f. During the first phrase of the second part, the corners forming *Diagonal 1* change places, making sure that the transition lasts the same amount of time as the musical phrase (see Fig. 3). Furthermore, the rule is that the two corners have to always face each other, and so when they meet and pass each other in the middle, they have to turn round at the same time and dance backwards for the rest of the movement.
g. In the second phrase, the corners making up *Diagonal 2* repeat this movement.
h. In the last phrases of the dance, the corners spin round in the company of parallel A, first with the right hand and then with the left, forming a chain (see Fig. 4). And then the corners spin round together with parallel B, first with the right hand and then with the left, forming a chain (see Fig. 5).

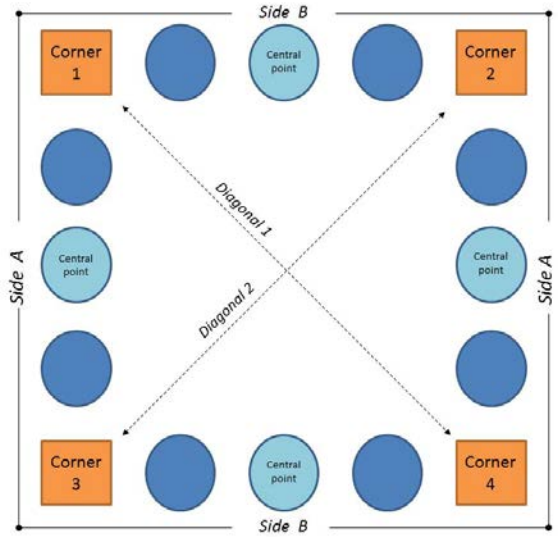


Figure 1: Initial position with the elements for this dance

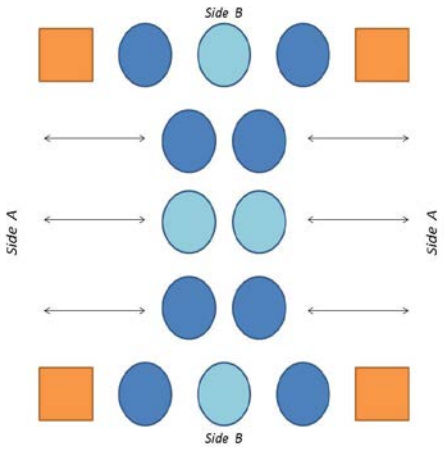


Figure 2: First movement

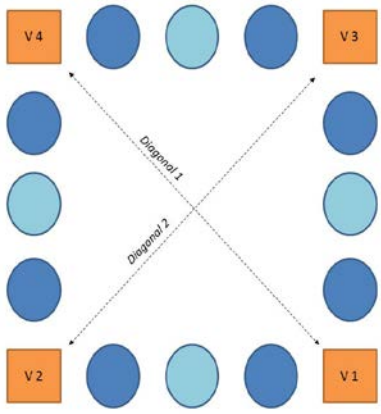


Figure 3: Second part movement

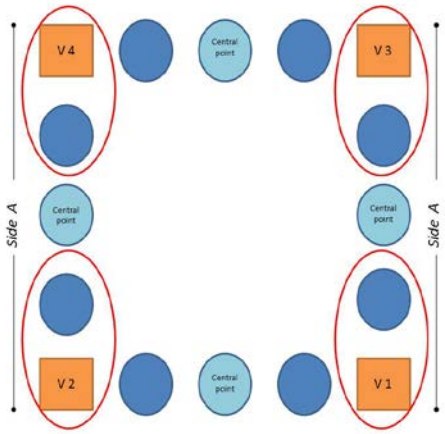


Figure 4: Chains in penultimate phrase

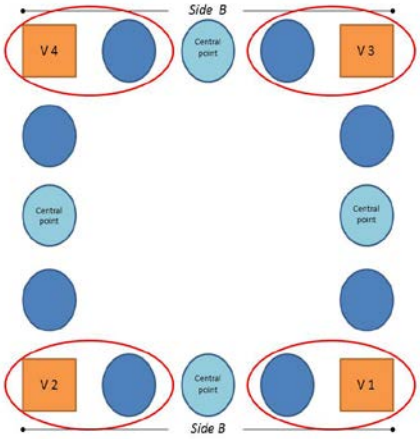


Figure 5: Chains in last phrase

Variations

1) The dance steps (point 4) can be modified. Depending on the degree of expertise, you can decide whether to teach the ternary rhythm (called *pas de ball pla creuat* in Catalan) and the binary one (*galop*), or you can keep to simple movements following the beats of the bar.

2) Depending on the number of pupils you can modify the dance and geometric shape. If the form is a rectangle, you can refer to the sides as short and long. This can help first- and second-year primary school children to understand the task.

N.B.
You can find a summary of a learning experience using this activity (expanded) in Palou, Casals and Prat (2016).

What is the LCM of the numbers 2, 3 and 5? Let's beat it out!!

Original idea: Cristina González-Martín

This activity, which brings together music and mathematics, consists of finding the lowest common multiple (LCM) of 2, 3 and 5 using body percussion.

The following content is studied:

Competencies

- Aspects of mathematics competency: reasoning, testing and making connections
- Aspects of music competency: listening and performance.

Key points

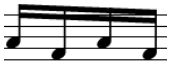
- Maths: numbers, multiples (and divisors), least common multiple (LCM)).
- Music: timbre, rhythmic reading, beat, rhythmic regularity and precision, body percussion.

Description of the activity

1. Start working with the number 2 (see example the end of the activity). You will present students with a body percussion score with thirty beats. You explain to them that each symbol corresponds to percussion on a different part of the body as follows:


Pat chest, right hand, and then left hand.


Clap hands.


Pat thighs, right hand, and then left hand.


Stamp with foot.

2. If the level of the students allows it, you can perform a rhythmic reading of the score, where the beats have to be marked with numbers, so as to always match up the clapping with the multiples, in this case the number 2 (2, 4, 6, 8 ... up to 30). If

students cannot read the score, teach them by imitation, which develops rhythmic memory. Once learned, apart from performing the body percussion the pupils have to count to 30 (following the beat).

3. Next teach the body percussion for number 3, where the multiples of number three (3,6,9 ... up to 30) also match up with the clapping.
4. Divide the class into two groups, facing each other in two lines. One group performs the body percussion for number 2, and the other, for number 3. Whenever there is a common multiple they will clap hands at the same time. The first time they do it, they will find the LCM of 2 and 3, i.e. the number 6. They can also discover other common multiples (12, 18, 24, 30).
5. They then learn the body percussion for the number 5, where the common multiples also coincide with the clapping. Facing each other in two rows they can identify the LCMs of 2 and 5 (10), and 3 and 5 (15 and 30).
6. Finally, we arrange the pupils in three lines, two facing each other and one at right angles to create a C-shape, and each line performs the body percussion of one of the numbers (2, 3, 5). When all the children clap at the same time they will have found the LCM of the numbers 2, 3 and 5 (30). Remember that they must count to 30 following the beat to find out what number all three lines have reached when they clap at the same time.

   You can make a graph of the rhythms where the three superimposed body percussion scores can be observed to find out what numbers coincide when clapping, thus identifying the LCM of the numbers 2, 3 and 5.

Variations

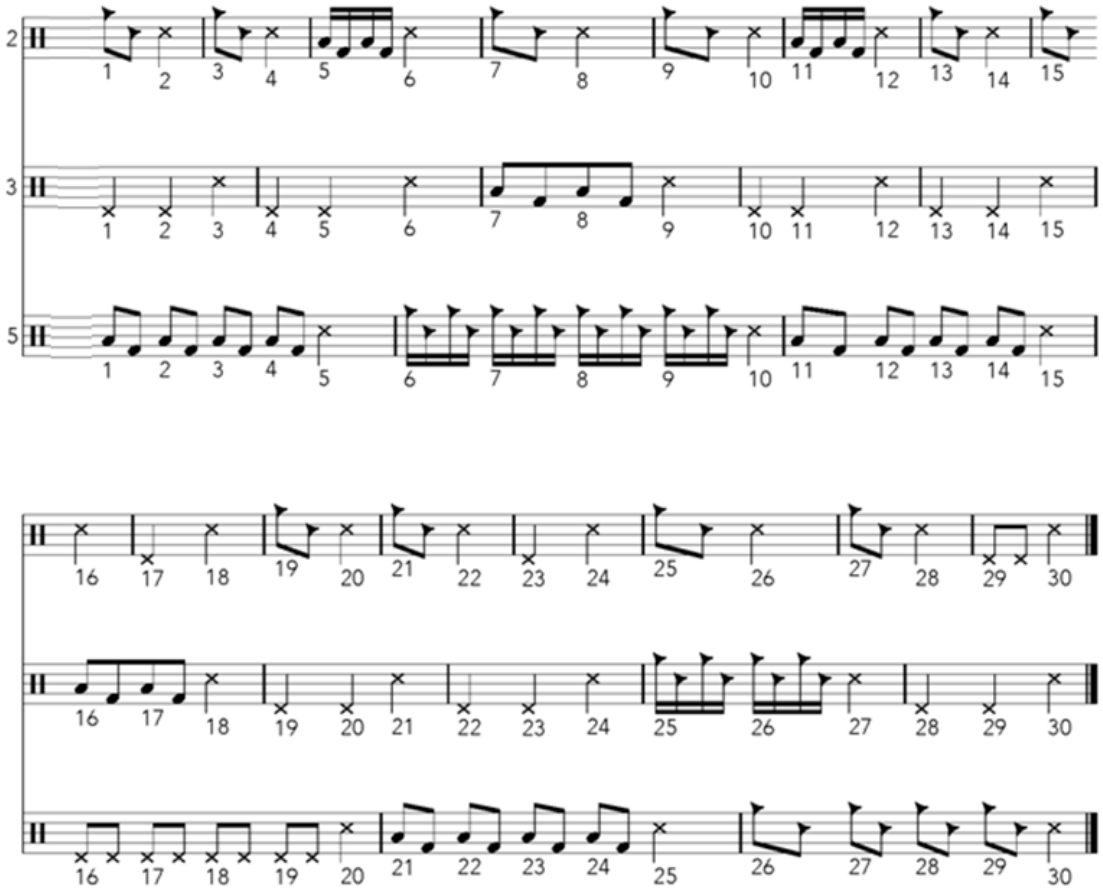
1) A variation of this activity is to use the notes G, E, C and C instead of body percussion. In this way pupils also work on the concept of the chord.
   The procedure is the same but the score is changed to include the notes, so instead of making the clapping of hands the common multiples will always coincide with the note G.

2) A second variation is that everyone stands in a circle and makes a chain (one pupil begins and is followed by the person on their right and so on). Each pupil marks the rhythm of a beat while saying the number it corresponds to. If we mark the rhythm of number two, then we realize that those who have clapped their hands are 2, 4, 6, 8, etc. If we repeat the activity with the rhythms of 3 and 5 (always starting with the same person), we discover that the only person who clapped their hands three times (with 2, 3 and 5) had the number 30 (which is the LCM of these three numbers).

N.B.

For further information on this activity, see also González-Martín, Pérez-Moreno and Prat *(in press)*.

Example of body percussion according to the described activity:

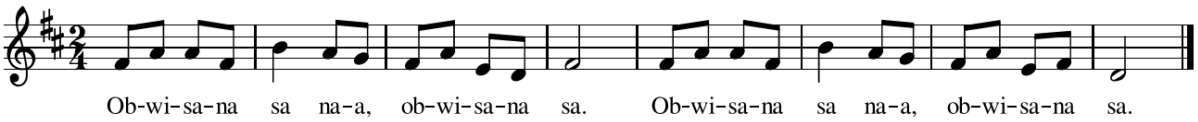


Obwisana

Original idea: Markus Cslovjecsek

This activity, which brings together music and mathematics, consists of singing a song, in this case *Obwisana*, to the accompaniment of the body, following the beat that will provide the context for the mathematical problems that have to be solved.

The proposed song, *Obwisana*, is a work song from Ghana. A rough translation is "Oh granny, I've hurt my fingers on a stone":



Competencies

- Aspects of mathematics competency: solving problems, reasoning and trial and error
- Aspects of music competency: listening and performance.

Key points

Music: listening to and performing a song with gestures.
Mathematics: least common multiple.

Description of the activity

1. Explain that in Ghana, while the men toiled at breaking stone, they would sing a song. The teacher sings the song and the pupils gradually join in until everybody knows it. It is important to note that the phrases end with a minim, and so you need to lengthen the note to two beats (i.e. to avoid making a crotchet and a rest).
2. From time to time, the workmen would brush the dust off their hands. Therefore, the movement of slightly bending the knees and slapping hands on thighs is introduced. This movement is repeated while we sing the song (making the slap coincide with each beginning of a bar → the pupils follow the beat of the bar).
3. Once they had dusted off their hands, the workmen would rest for a moment and then straighten their backs (the pupils place their hands on their hips). Then the song is sung alternating the two movements (cleaning the dust off their hands the first time the bar is heard, and hands on hips, the second time).
4. Finally, it is explained that the workers used to complain about the hard work they had to do and that they often ended up getting hurt. And they used to express this by crossing their arms in front of their chests. And so this third movement is included: the song is sung alternating the three movements and always following the beat.
5. After singing the song with the accompanying gestures, the children must realize that after the last minim in the song they are not in the third movement (the last movement of the sequence of gestures), but the first. They will wonder why (the song has a number of beats that are not a multiple of 3) and they should find out whether this always happens. And so you should propose singing the song repeatedly, as many

times as necessary, until the end of the melody coincides with the sequence of gestures. The pupils will discover that they need to sing the song three times.

6. Finally, the pupils again sing the song three times with the accompanying gestures. They are asked to count how many times they make the gesture of putting their hands on their hips. They discuss how many times (they often don't agree). When they all agree that it was 16 times, they are asked how this is possible if they sang the song three times and 16 is not a multiple of 3. They have to think it out and find the solution: each gesture is repeated 16 times, 16 x 3 = 48 and 48 is a multiple of three (gestures) and 16 (the number of beats in the song). In fact, it is the lowest common multiple (LCM) of 16 and 3.

Variations and complementary activities

1) You can lengthen the sequence (four gestures, five gestures...) to show that the solution is always the LCM of 16 and the number of gestures performed. A visual representation and manipulatives make good complements to help all the pupils finally understand the process.
2) Instead of *Obwisana*, you can use other songs and other sequences of gestures, looking for a number of beats and a number of gestures that are not many multiples of each other. For example, a musical phrase with eight beats and six gestures. In this case, the solution would be to perform the song three times (24 beats).
3) Later, you can explore the connection with any typical mathematical problem (for example, a statement that explains that yoghourts packed in boxes of 16 have to be repackaged in boxes of three so that none are left over. To solve the problem, apart from using the usual resources (graphics, manipulatives), you can resort to the use of musical phrases combined with gestures to find the solution.
4) On a musical level, you can go deeper into various aspects such as the following: performance of the song; structure of two phrases (antecedent-consequent); performance of the melody on a xylophone or similar instrument (you can go down a tone to C major); and accompaniment with percussion instruments of your choice, etc.

Making angles while moving to the music

Idea original: Albert Casals

In this activity, we work on musical structure, the angles and the expressiveness of music while choreographing a short musical fragment.

Competencies

- Aspects of mathematics competency: representation
- Aspects of music competency: listening (and performance).

Key points

- Mathematics: angles.
- Music: structure and phrasing, physical theatre.

Description of the activity

1. In a classroom or other large space, the teacher stands in front of the pupils. They will also be standing up and must be able to see the teacher. The starting position is with arms hanging down in the resting position.

2. When the music begins, the teacher makes deliberate gestures with parts of the body (it's easiest with the arms) so as to gradually create a short choreography. This helps the children to understand the musical structure while making certain angles (90º, 180º, 45º, etc.). All the pupils follow and imitate the proposed choreography (with a mirror effect). You can find a specific example at the end of the description.

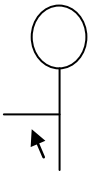
3. Once completed the choreography, the students divide into groups to discuss and represent the angles that have been marked out most clearly.

4. Finally, using the same music or any other piece with a simple structure, the pupils are given the musical task of extracting and analysing the musical phrases, and they decide how to represent the structure using body movements that make angles (individually or working together).

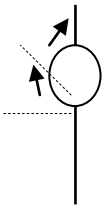
Example of music and choreography

Using the beautiful music of the Intermezzo in Suite No. 2 from *Carmen* by G. Bizet, in the version found at: https://www.youtube.com/watch?v=eChLCFAGyx0, we propose the following movements:

a. Following the introduction by the harp, when the flute comes in, the pupils slowly raise one arm up to shoulder height, where they stop to make the angle (approx. 90º).
[0'10" – 0'27" in the chosen version]

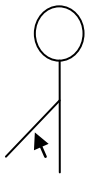


b. They continue raising this arm until their fingers are pointing straight up (180º), and they stand on tiptoe at the high point of the phrase [0'48"] and then slowly drop the arm to the starting position.
[0'28" – 0'53"]



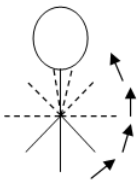
c. They make the same movements (steps a and b) with the other arm while the clarinet leads the melody.
[0'54" – 1'37"]

d. They raise the first arm again, but now only to 45º, and then return it to the starting position.
[1'38" – 1'45"]



e. They repeat the same movement with the other arm.
[1'46" – 1'54"]

f. They raise both arms at the same time, always slowly, until they bring their hands together over their heads (360º).
[1'55" – 2'21"]



g. They gradually lower both arms towards the starting position, but stopping at two points [2'32" and 2'41"]. The first point is when the arms are horizontal, forming a cross with body (90º + 90º = 180º). The second point is when each arm is positioned at 45º to the body's longitudinal axis (i.e. making 45º + 45º = 90º). The starting position is reached at 2'51".

h. Finally, the children cross their arms slowly in front of the body and finish by placing opposite hands on the shoulders (left hand on the right shoulder and vice versa). With the last notes of the harp, they may end by lowering their heads as if they wanted to rest their chins on their chests. Then they keep absolutely still while listening to the final *pizzicato* on the strings.

Variations and extensions

1) Pupils can carry out a task of observation and also measure the angles using a protractor. While some of them perform the choreography, others take photographs of the different movements, thereby capturing the angles being made. The photos can be screened or printed out and the angles made in the choreography observed and/or measured.

2) Given that the movements performed during the choreography are made with both arms, there are moments of axial symmetry (movements *f* and *g* in the choreography suggested at the end of this description), as indicated by the drawing. This provides an opportunity to focus pupils' attention on this content and modify the choreography to work on other types of symmetry like the central one: with the axis of symmetry formed by the arms stretched horizontally at 90º. To create a plan of symmetry, movements should be made above this axis and then repeated identically below it, so that the point (in this case the hand), the image (in this case, the other hand) and the centre form part of the same line. First above, then below, though depending on the age of the pupils, both could be explored at the same time, as shown in the illustration.

3) You can use the same music to work on easier content. Instead of studying angles and symmetry, pre-school and first- and second-year primary school children can explore the situation of the space. Thus, using the same phrasing as indicated above, the activity now begins by arranging the whole class into two separate circles, one with more students (approx. 2/3) and the other with less (approx. 1/3 students). At this point you have an opportunity to work on the form of the circle: the circumference. Taking this starting point, a possible sequence is as follows:

   - 8 introductory beats marked by tapping with the heels and with hands on hips.
   - 16 beats; holding hands, the two circles (circumferences) turn clockwise.
   - 16 beats; holding hands, the two circles turn anticlockwise.
   - 8 beats; the small circle breaks up and re-forms inside the large circle.
   - 16 beats; the two circles turn clockwise with the pupils holding hands.
   - 16 beats; the two circles turn anticlockwise with the pupils holding hands.
   - 8 beats; the large circle breaks up to form a straight line or row (one child behind the other). The small group watches.
   - 8 beats; the small group advances to create a line in parallel to the longer line on its right side, while others tap out the beat with their heels, and with hands on hips.
   - 8 beats; the two lines move forward.
   - 8 beats; the two lines move backwards.

- 8 beats; the line made by the small group positions itself perpendicular to the longer line, looking for a imaginary point of intersection between the two, approximately at the midpoint of the longer line (as **I-**)
- 8 beats; keeping straight, the shorter line moves perpendicular to the other line to reach one end so that the two lines form an **L** shape
- 8 beats; the children stay in their places while tapping out the beat with their heels and with hands on hips.
- 8 beats; they all hold hands, the children at the ends too (who seek each other out), thereby breaking up the lines and creating a circle that turns until the music ends.

N.B

We have highlighted some of the mathematical concepts that are clearly reflected in this activity and are likely to be worked on.

Fractions and rhythm

Original idea: Montserrat Prat and Cristina González-Martín

This activity aims to bring to light the connection existing between fractions and rhythmic notation (semibreves, minims, crotchets, quavers and semiquavers) using visual material and manipulatives. The material can be adapted to different rhythmic notation. And, furthermore, it can be used to compose short pieces.

Competencies

- Aspects of mathematics competency: problem-solving, representation and connections
- Aspects of music competency: listening, performance and composition.

Key points

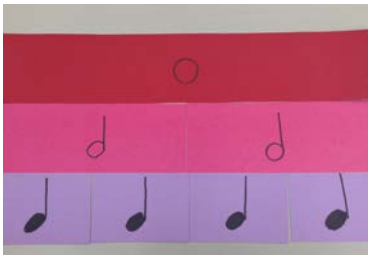
- Mathematics: fractions (as part of a whole).
- Music: rhythmic notation

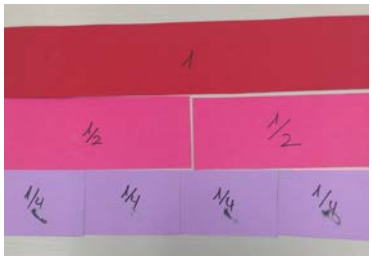
Description of the activity

We propose two different activities, plus an introductory activity that serves to familiarize the pupils with the work material. The two activities can be carried out together or separately, depending on the level, how much the pupils know, and how far you want to explore the theme. On the other hand, it makes no sense to only do the introductory activity (which lacks the component of "making music").

Introductory activity

The material used in this activity has both mathematical and musical aspects. Thus, on the one hand there is the representation of examples of rhythmic notation and, on the other hand, the fractions they represent.





1) The first step is for the teacher to establish what note value will be represented by the unit, either the crotchet (usually equivalent to one beat), the minim (which is the unit of 2/2 time and often used to mark the accents in the bar) or the semibreve (which is assigned the value of 1 in the conventional writing of bars).

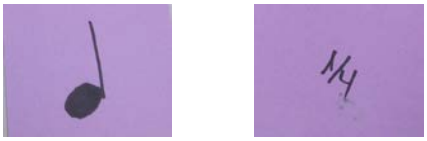
2) Once you have chosen what note value will comprise the unit, thereby representing the total as a fraction, a 1 is drawn on one side of the card. In this case, to explain the activity, the note value chosen as the unit will be the semibreve (𝅝) and it should be drawn on the other side of the card.

Then the teacher performs the semibreve to establish the length of the unit on which the following note values will be based.

3) The teacher then takes two cards half the length of the card representing the unit. Each of these cards will make up ½ of the total and, therefore, each one corresponds to a minim (𝅗𝅥). Again, the note should be played to make sure the two minims are equal to one semibreve.

N.B. It is the teacher who decides whether the pupils make the card that represents ½ of the total, or whether the pupils only have to search among the cards of different length to find the one equivalent to ½ (𝅗𝅥).

4) Then take a card half the length of the card labelled ½. The pupils will see that in order to equal the length of the card labelled 1 (𝅝), they need four of these new cards. And so each of the new cards represents ¼ of the total, and musically they correspond to crotchets (♩), as you should demonstrate by performing them.



If you do this activity as a process of problem-solving, the pupils will discover the relationship between the various note values and clearly understand that the minim is half a semibreve (since two minims are needed to match the length - both in the

visual and aural senses – of a semibreve (𝅝); 4 crotchets (♩♩♩♩) are needed to equal the length of a semibreve, and therefore a crotchets corresponds to a ¼ semibreve; and two crotchets are equal in length to one minim, and therefore half a minim is a crotchet.

5) To make sure that pupils understand these aspects and to check that they are able to keep to the same beat while following the different rhythms on the cards (either with body percussion, voice, instruments), you can do the following:

   - Organize the pupils into small groups (each of which has a set of the material discussed above). Using the card displaying the semibreve, ask them to prepare different rhythmic combinations equivalent to the semibreve in length and subsequently perform them.

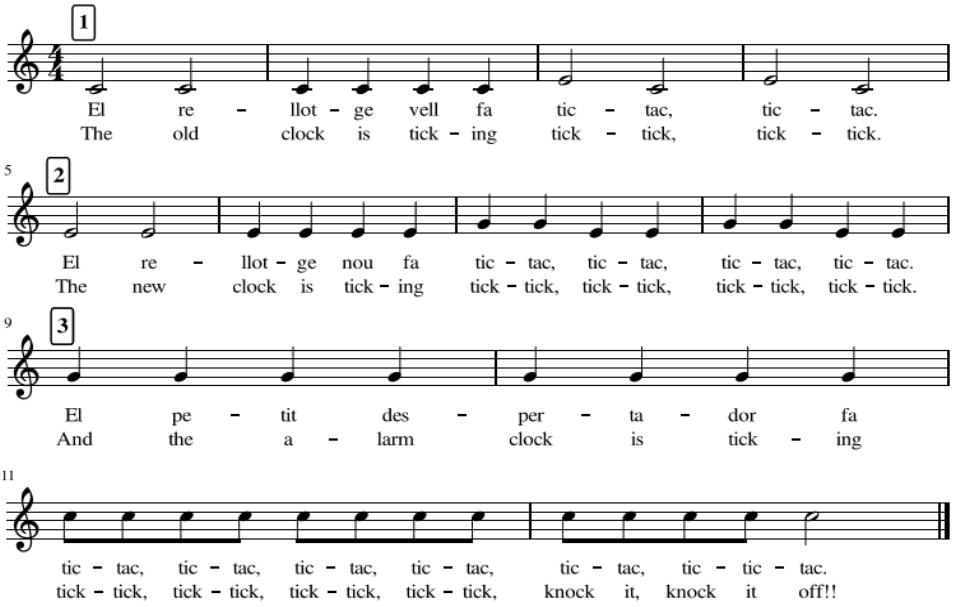
   - It is interesting to suggest to pupils that, individually or in pairs, they create short compositions with the different, small note values they have studied. If you wish you can set certain conditions (four bars of 4/4, for example). Then, the compositions can be performed in front of classmates.

At the end of this first activity it is worth asking the children to reflect on the fact that this process of problem solving, representation and connection enables them to discover that fractions have no specific value in themselves and that their value depends on the whole (vision of fractions as part of a whole). And, therefore, ½ a semibreve (𝅝) is a minim (𝅗𝅥) and consequently has a specific length (sound duration), different to ½ a minim (𝅗𝅥) which is a crotchet (♩).

After this introductory activity, we propose the following two activities:

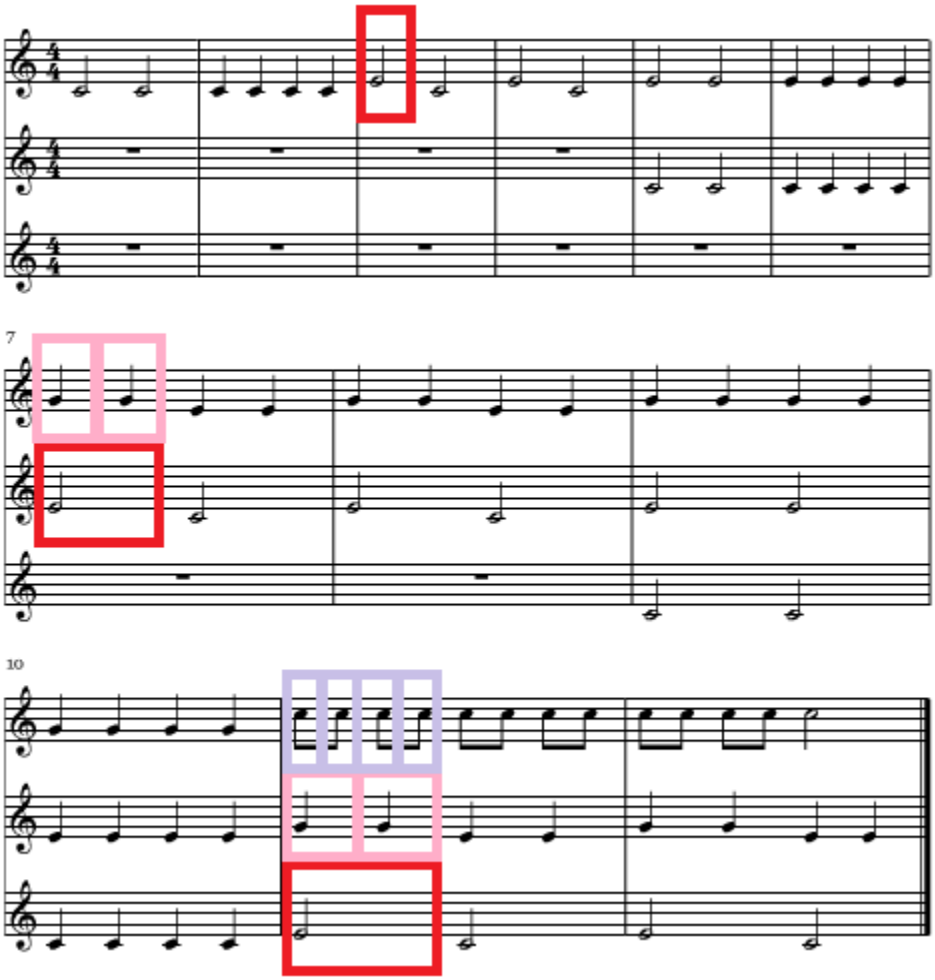
Activity 1

In this activity the unit will be represented by the minim. The rhythmic performance is based on the song *El Rellotge* (the clock):



The process is as follows:

1) The pupils are taught the song.
2) Once they have learnt it, they are asked to clap their hands to the rhythm of the song while they sing it (or they can also be asked to walk around the classroom in time to it). The pupils should be informed that depending on the clock type (old, new, or alarm clock) the "tick-tocking" is represented by a different note value (minim, crotchet and quaver).
3) At the beginning of the following session, the pupils practice and consolidate the song. They are then split into three groups and sing the song in canon. When they sing it in canon the equivalences between the "tick-tocks" of the three types of clock become perfectly clear, which mathematically would form a unit (the old clock), half (the new clock) and a quarter (the alarm clock). To make it even clearer, each group is ask to clap in time to the rhythm while singing the song. Thus, if they watch their classmates during the final part of the canon they will notice the equivalence on observing how they move their hands or body (if you have decided they should walk to the rhythm instead of clapping to it) and on listening to what is happening melodically and rhythmically.
4) Once carried out in practice, the pupils can be taught the score of the canon, where you will have indicated (according to the colours of the cards from the previous activity) the different equivalences:



As a complementary activity, they can be given the score and asked to look for and mark any other equivalences they find there and then make an effort to indicate them aurally by singing or clapping. Or conversely, they can look for moments when the voices sing different rhythms to each other, discovered their relative value (half/double) and check it out in the score. They can establish links between the unit, ½ and ¼, or between the unit and ¼, etc. It is also interesting to note that ¼ is half of ½, as this makes it clear to the pupils that in this case the fractional values are all relative to each other and that the value depends on the reference unit.

Activity 2

In this activity the unit will be the crotchet. The rhythmic performance is based on the audition of a fragment of the Canon in D major by Pachelbel. One of the many existing versions can be found at: https://www.youtube.com/watch?v=jJRdLZyOU4w . During this activity, and because the whole piece is too long, you only need to use the first part, looking for a suitable end point (this could be at 1'27" or 1'56" in the proposed version).

The steps taken are as follows:

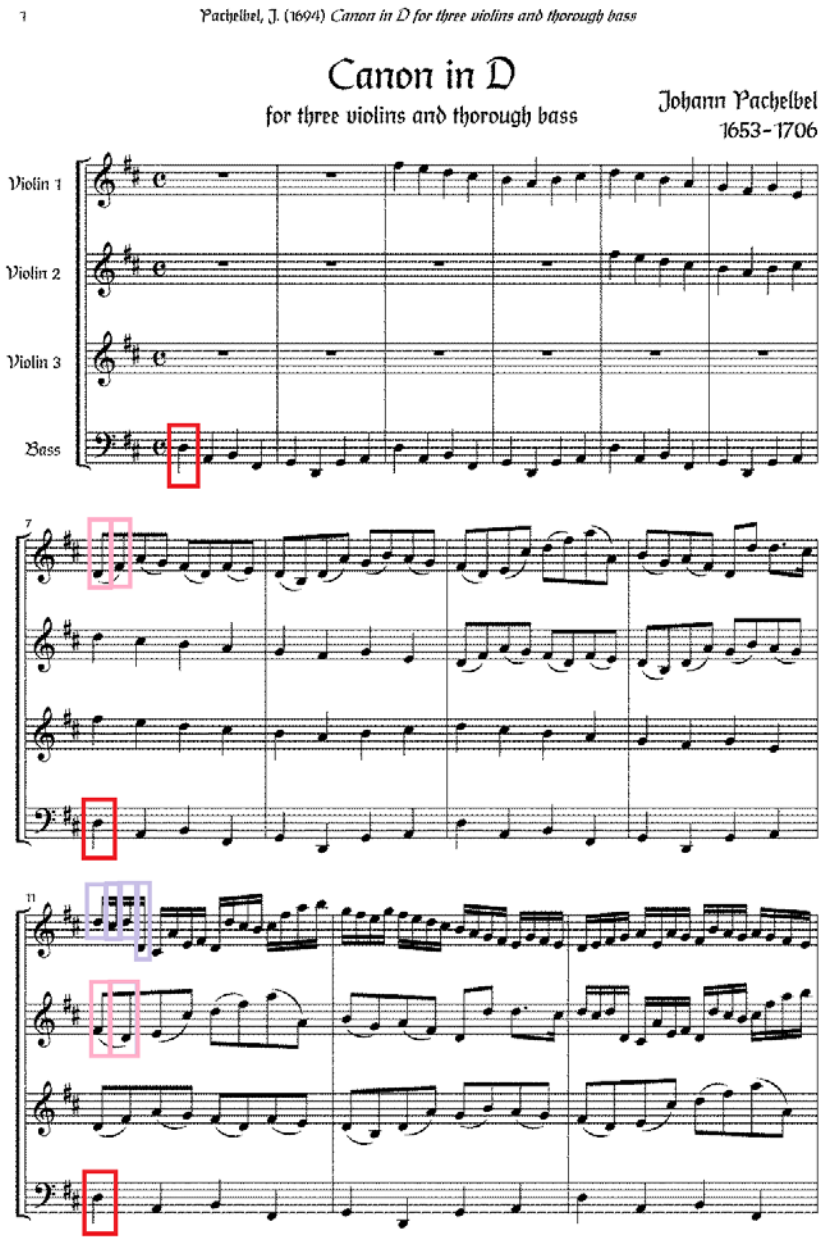
1) Pupils listen to the piece while clapping or walking to the rhythm of the basso ostinato, which is always constant and uses crotchets (in this case the note value representing the unit).

2) In a second listening, the whole group follows the rhythm of the first voice together (you can ignore this step if the group is strong musically).

3) For the third listening the class is divided into two. While some follow the rhythm of the ostinato, the others follow that of the first voice. In this way pupils are able to establish the rhythmic relationship (equivalence) between these two voices. First, they all do the same thing until the first violin begins to double up, i.e. play at a rhythm that is twice as fast (quavers), which means that it lasts ½ the time of the bass (crotchets). And later it will end up playing ¼ length notes (semiquavers) compared to what the cello is doing.

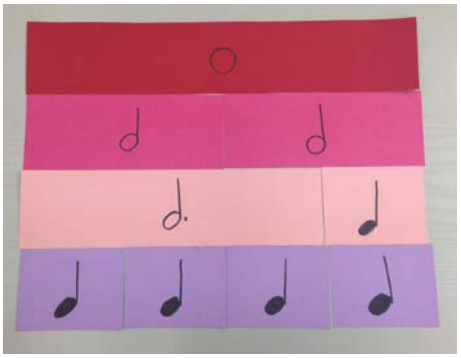
If the group has a good level, you can continue with two or even three voices in canon, apart from the bass. The same cumulative procedure as explained above should be followed and finally the class is split into three or four groups (as appropriate), each of which will follow one of the voices. In this way a point is reached where equivalence is produced between the unit, the ½ and the 2/4 simultaneously. Apart from hearing it, the pupils can see it represented physically in the way you have chosen to do so.

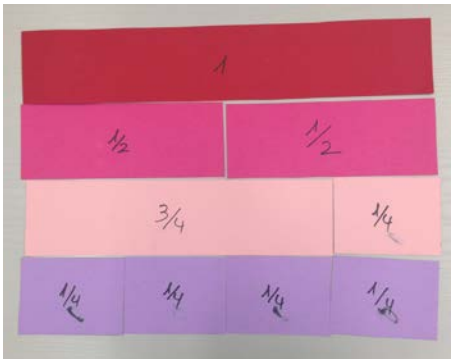
Finally, the same as in activity 2, you can teach pupils the score of the canon, where the different equivalences will be marked (using the colours on the cards from the introductory activity), and they can listen to and follow the score they developed as a practical activity



Variations

1) You can prepare cards representing 3/4 (see the image below). This allows you to introduce musical figures with a dot, where the dot represents half the value of the figure it accompanies.





2) You can play at creating small equalities (equations). For example, what combinations are equivalent to a semibreve + a minim? If you then make two groups and one group performs the semibreve and the minim (6 beats of a crotchet) while the other performs its supposed equivalent, they can check whether it is correct.

4. Suggested reading for further interest

Aside from this booklet and the EMP-M project training course it is linked to, you have a website (http://maths.emportfolio.eu) at your disposal where you can download all the project materials, articles and documents. In this respect, particular mention should be made of a **Teachers' Manual** (Mall et al., 2016), which includes some further activities, and **reviews of the state of affairs** (Hilton et al., 2015; Saunders, Hilton & Welch, 2015). Other material has also been prepared in each participating country and appears in the corresponding European language.

To find more ideas for activities, we also recommend some of the many **books specialised** in the combination of mathematics and music:

- An, S.A., & Capraro, M.M. (2011). *Music-math integrated activities for elementary and middle school students*. Irvine, CA: Education for All.
- Cslovjecsek, M. (ed.) (2001/2004). *Mathe macht Musik: Impulse zum musikalischen Unterricht mit dem Zahlenbuch*. (Vols. 1-3). Zug, Switzerland: Klett und Balmer
- Nolan, Karin K. (2009). Musi-Matics! Music and Arts Int*egrated Math Enrichment Lessons.* New York: Rowman & Littlefield Education/MENC.

Lastly, we present a list of **bibliographical references** that, from many angles, can help you to go deeper into the subject.

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